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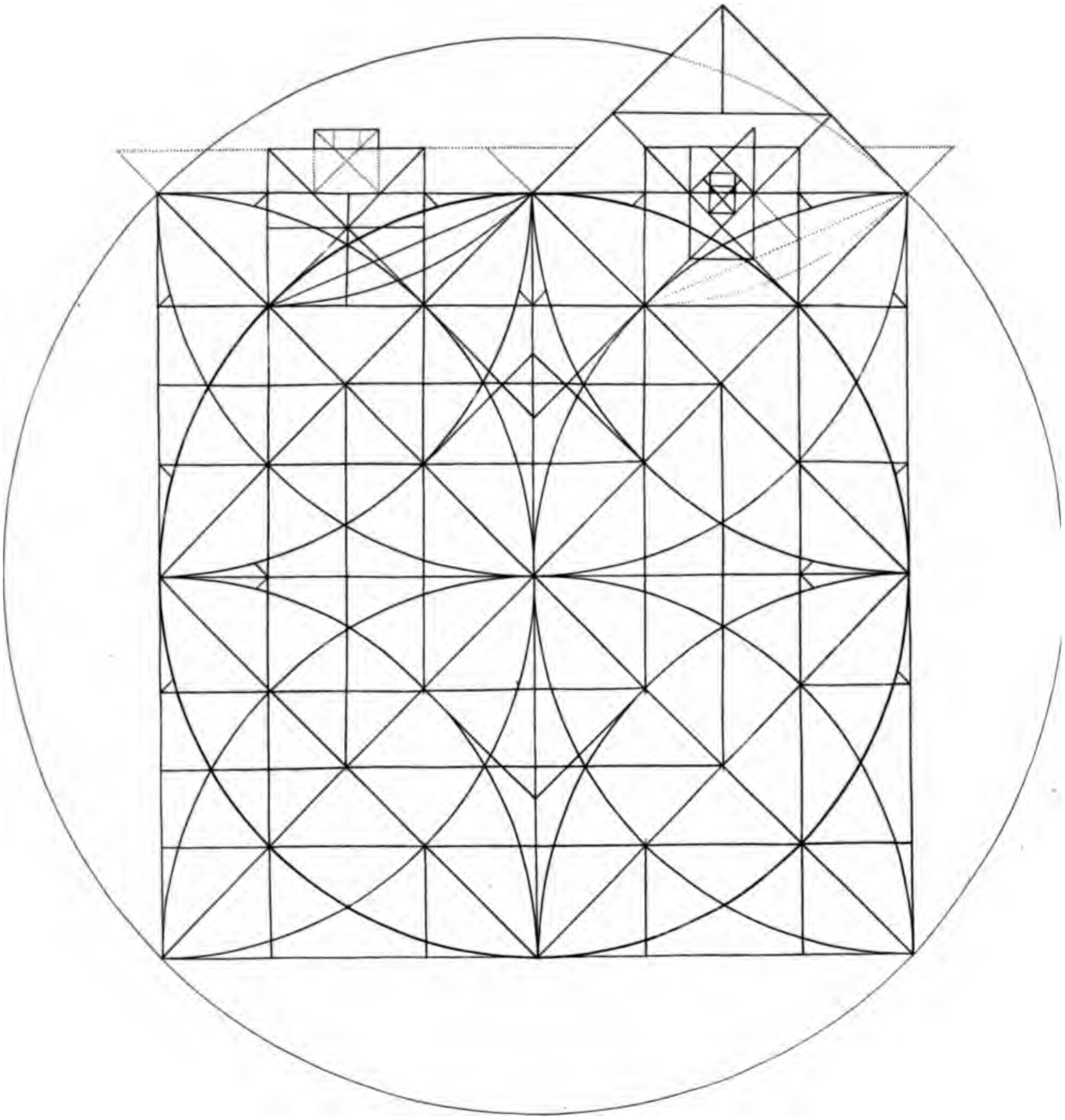
DIAGRAM

SHEWING THE CORRESPONDING COMPARTMENTS

OF THE FIGURE.



THE

Circle Secerned from the Square,

AND

ITS AREA GAUGED IN TERMS OF A TRIANGLE COMMON TO BOTH;

ALSO

An Original, Simple, and Exact Method pointed out

OF

APPROXIMATING AS CLOSELY AS POSSIBLE TO THE NUMERAL VALUE OF THIS TRIANGLE, AND THE CONSEQUENT VERITABLE CONTENT OF THE INSCRIBED CIRCLE, IN RELATION TO ANY GIVEN SQUARE.

BY WILLIAM HOULSTON, ESQ.,
(FORMERLY SUB-COLLECTOR OF HER MAJESTY'S CUSTOMS, PORT MORANT, JAMAICA.)

Ἴσμεν γὰρ οὐδὲν τρανές, ἀλλ᾽ ἀλῶμεθα.
SOPHOCLES.

"As Phœbus to the world, is Science to the soul.
And Reason now through Number, Time, and Space,
Darts the keen lustre of her serious eye,
And learns from facts compared, the laws to trace,
Whose long progression leads to Deity."
BEATTIE.

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TO THE

RIGHT HONOURABLE LORD BROUGHAM AND VAUX,

AS A TOKEN

OF THE AUTHOR'S ADMIRING APPRECIATION OF HIS LORDSHIP'S DISTINGUISHED MERITS,

IN ADVANCING

THE INTERESTS OF HUMANITY,

BY PROMOTING THE PURSUIT OF SCIENCE,

THIS

BRIEF EXPLICATION OF THE NUMERICAL CAPACITY OF THE CIRCLE

IS RESPECTFULLY INSCRIBED.

BIBLIOTHECA·BODLEIANA·

Advertisement.

> "Measured are the skies,—
> Stars are detected in their deep recess,—
> Creation widens." YOUNG.

THE slight contribution to Mathematical Science, made known to the public in the following pages, acquires no slight degree of importance from the great historical interest attaching to the fact of its clearing up what had remained a *vexata quæstio* ever since the period when first the measurement of the circle's area became an object of investigation to the scientific inquirer, notwithstanding that many first-rate mathematicians of every age had applied their skill in endeavouring to arrive at a full resolution of the subject.

It is, consequently, with no slight satisfaction, and some feeling of pride, that I find myself,—after much perplexity, and laborious research,—the fortunate discoverer of such a method of solution, as will, I conceive, commend itself to the judgment and capacity of even ordinary intelligence. By first gauging the figure perfectly, in terms of itself, and then ascertaining the numeral value of the gauging terms, it detects any error that may lurk in less perfect methods of approximation; and shews, clearly, how to state by numbers the true content and quadrature, as nearly as the *nature* of numeral representative characters will admit.

But my feelings of exultation, at being thus enabled to settle this long disputed question—as I trust *for ever*,—are most gratefully heightened, by the reflection that humanity at large must benefit by my success,—prospectively at least,—if what I have heard reported be indeed true, that the asylums of several countries have among their inmates many patients who have lost their reason through perseverance in unsuccessful efforts to solve this peculiar problem.

With respect to the method itself, I must acknowledge, on first apprehending to what a manifest divergence of measurement from the approximation ordinarily received as correct, it infallibly leads,—shewing a difference somewhat less than the seventh part of a mile in the distance of a thousand miles,—I felt a natural reluctance to come with it into publicity, and not the less so, from my experience of very many fallacies which beset the subject and blind the mental vision of the inquirer; but, finally, the evidence of its truth appears to my mind so completely a demonstration, and the process, like the fixing of the egg by Columbus, now that it is arrived at, seems so simple a matter in itself, that I am induced to challenge other minds to its investigation, nothing doubting but that by applying their best intelligence to the subject they must arrive at my conclusion; and that they will consequently then exonerate me from the seeming arrogance of venturing to adopt the well known saying of the Syracusan Sage,—so far as discovery is concerned,—and in my enthusiasm exclaim, *"I have found it!"*

Ennisville, Beaumont, Jersey. W. H.
January, 1862.

Contents.

Section. **Page.**

DIAGRAM SHEWING THE CORRESPONDING COMPARTMENTS OF THE FIGURE.

I. AIM OF THE AUTHOR 4
II. PRELIMINARY ADMISSION 6
III. CORROBORATIVE CONSIDERATIONS 7
IV. ESTIMATES OF OTHERS 8
V. ANALYSIS OF THE DIAGRAM 8
VI. COINCIDENCE OF EXTERNALS WITH INTERNALS 10
VII. CAPACITY OF CERTAIN COMPARTMENTS 11
VIII. CONTEST OF INIGO AND OUTIGO 12
IX. MEASURE OF THE TRIGON AND ITS PARTS 13
X. APPLICATION TO A CASE 14
XI. AREA OF SCHOOLS AND AUTHOR CONTRASTED 15
XII. CONCLUDING REMARKS 18
ANALYSIS OF THE SEDECIMAL-TRIANGLE *in Extenso* 20
THE RESULT IN THE DECIMAL SCALE 22

[ENTERED AT STATIONERS' HALL.]

The Area of the Circle Computed

from its Compartments.

SECTION I.

AIM OF THE AUTHOR.

> "Siste! quò præceps ruis?"
> "Rapior, et, quò, nescio;
> Sed rapior."—SENECA.
>
> "In vain you damp the ardour of my soul,
> 'Replied Euryalus', it scorns controul."—BYRON.
>
> 'Αλλ' ἡ βία γὰρ ταῦτ' ἀναγκάζει με δρᾶν.—SOPHOCLES.

WHAT I propose to grapple with is a Problem that has tasked the ingenuity of the human intellect for ages;—the discovery of the true ratio that exists between the space occupied by a square, and the space occupied by the largest circle that can be inscribed therein.

> "Since neither now nor yesterday began
> These thoughts, which have been ever, nor yet can
> A man be found who their first entrance knew."
> EMERSON.

The solution of a point with respect to one diagram, will be a solution as to all similar figures; since all geometrical forms of a class, possess the same properties, whatever may be their relative size. The chase after so fugitive an element, if commendable in days of yore, can, I imagine, hardly be a just butt for censure in these latter days, when Science reckons up so many conquests that surprise us.

> "And let us mind, faint heart ne'er wan
> A lady fair;
> Wha does the utmost that he can
> Will whyles do mair."
> BURNS.

The laughing world's opprobrium attaching to adventure and failure, on so hackneyed a subject, I must brave as best I may,—

> "Fix'd statue, on the pedestal of scorn."
> BYRON.

But may the fates reserve for me a better lot in the realization of success,

> Si verò εὕρηκα in fatis est tandem ut canendum sit!

and how compensating a consideration is that of eventual triumph! for where is the gifted individual that can fully foresee the many important advantages that would accrue, in certain of the Arts, and to Astronomy, Navigation and kindred Sciences, dependent on calculations for the comprehension of their phenomena! Besides, I am not insensible to the promptings of ambition's titillating hand;—

> "I own I labour for the voice of praise,
> For who would sink in dark oblivion's stream!"
> WOLCOT.

Wherefore, let me evoke energy in my feebleness of spirit, by the animating.apostrophe of the classic:

> "Anime, quid rursùs times? audendum est: age!"
> "Funditùs cuncta erure!"
>
> SENECA.

and if, after all, I lapse, "magnis ceciderim ausis;" and may the poet's deprecation then bestead me:—

> "Still let some mercy in your bosoms live,
> And if you can't applaud, at least forgive!"
>
> BYRON.

I am not so presumptuous as to imagine that I can state the exact contents of any square and its cognate circle, in the abstract digits of the Arabian notation,—each unit of which, if once employed in the Problem to represent a square of space, must continue to do so, or its constituent square parts; and most eminent Mathematicians have demonstrated the fact that these peculiar spaces are incapable of perfect representation by numbers, being incommensurable by the gauging square of the Arabian unit; but, that the ratio of the two figures can be exactly determined, and stated, in terms of their own constituent compartments, my aim will be to shew; and that, thus, through the demonstrable value, to a definite extent, of certain of these spaces, the contents of the figures in question—licet obtutum oculorum fugiant,—can, in all certainty, be approximated by numeral notations, unto any degree of exactness, that the patience of the calculator may choose to pursue the evolvement; each successive residue of space becoming, in the interminable process, O how minute to mortal ken!—

> "Faint, lessening, quivering, glimmering, now no more!
> Thus shall the sons of science sink away,
> And thus of beauty fade the fairest flower—
> For where's the giant who to Time shall say,
> 'Destructive tyrant, I arrest thy power?'"
>
> WOLCOT.

SECTION II.

PRELIMINARY ADMISSION.

> "Dispute it like a man!"—SHAKSPEARE.
>
> "He grants, objects, now questions, now replies."—BROWN.

YET I am bound to confess my conclusions are partially based on a seeming assumption of the identity, as to space, of two particular compartments generated by the construction of the diagram, the *one* being within the circle, and rectangular convex, in form; the *other* being without the circle, and, in form, rectangular concave; let us discriminate them, respectively, by the names of "Inigo" and "Outigo."

These items, in especial, *viderint doctiores*; and let them point out the fallacy of my inference, if fallacy there be in it; for very strong analogical considerations, and the continual harmonious compensations formed by intersections that pervade the figures, support the assumption in every respect.

This single *petitio* conceded, the ratio becomes simple to admiration, for only two correlated gauges, or terms of the figures, suffice to measure the whole square, and its interesting nucleus; twenty of the one, gauging its circumclasping corners; sixty of the same, along with sixteen of the other, gauging the embraced circle.

> "Adspice; namque omnem, quæ nunc obducta tuenti
> Mortales hebetat visus tibi, et lumina circum
> Caligat, nubem eripiam."
>
> VIRGIL.

SECTION III.

CORROBORATIVE CONSIDERATIONS.

"Naturam expelles furcâ, tamen usque recurret."—Horace.

"Age cannot wither her, nor custom stale
Her infinite variety!"—Shakspeare.

"Et l'œil qui nous instruit de sa beauté suprême,
En un circle brillant s'est arrondi lui-même."—Delille.

The symmetries of Nature are truly astonishing, when we duly consider them;

"Unica Naturæ facies illæsa nitescit,"
Gretton.

harmonious as the duplicate lustre of the fountain, "beauty to our eye, and music to our ear," and to be found everywhere; so lavish is the Creator in dispensing bounties!

If, ever, the even course of symmetry deviates, it is for a purpose, whether it be physical or moral,

"We look through a glass darkly, we catch but glimpses of truth;
But, doubtless, the sailing of a cloud hath Providence to its pilot,
Doubtless, the root of an oak is gnarled for a special purpose,
The foreknown station of a rush is as fixed as the station of a king."
Tupper.

Why do the concentric circles observable in a horizontal section of a timber-tree deviate from normal uniformity? but to register the erratic influences, and vicissitudes, of storms and seasons:

"The dash of clouds, or irritating war
Of fighting winds."
Thomson.

And why, in the pane of the rectangle, as shown in the diagram, do the analogous forms which I have named "Inigo and Outigo," deviate? but to fit in with the other compartments of the figure, and furnish the reciprocating spaces,—that in Section V. will be distinguished by name, as "the commuted trigon," and "semi-trigonal square,"—with an equivalent item, within or without the circle.

"Time, motion, number, sunshine or the storm
But mark varieties in Nature's form."
Darwin.

In minute figures it is fallacious to make our eye the judge of relative extent of space. We must find the truth out by reasoning on comparisons. Perfect accuracy in the construction of diagrams, it is well-nigh impossible to attain; for lines, geometrically considered, are boundaries which are invisible:

"Radiant as light that glimmereth above,
Soft as the ray in its celestial clime."
M.S. of "Alzara."

But ye lovers of Nature, who would view symmetries in perfection whether at early dawn, high noon, or later, when the panoramic dyes of sunset pale beneath the arching zone,—

"Just gaze one instant on the sky
Where glitt'ring worlds in azure lie."
M.S. of "Alzara."

"How sweetly mutable yon orient hues,
As morn's fair hand her opening roses strews;
How bright, when iris blending many a ray,
Binds in embroidered wreath the brow of day;
Soft, when the pendant moon, with lustres pale,
O'er heaven's blue arch unfurls her milky veil!"
Darwin.

SECTION IV.

ESTIMATES OF OTHERS.

> " Insulsissimus est homo, nec sapit pueri instar
> Bimuli." CATULLUS.

> "Thus far I've held my undisturb'd career."—BYRON.

FOR a circle whose diameter is unity .78125 is the decimal fraction that represents the area, according to Mr. James Smith's ingenious theory of a solution. The schools usually give it as .7854, or somewhat less than that; whilst my computation shews it to be .785535, *plus* a fraction interminate by the usual scale of number, as explained in Section I. But, for argument's sake, admitting that—with the eye alone as guide,—the compensating part taken into the circle, to form the commuted trigon, seems larger than the part which I have before referred to as left out of the compartment named the semi-trigonal square, by reciprocation; in that case,—since the two figures concluded by us to be only dissimilar in form, can no longer be equal to each other in space,—the content of the gross area ought to sum up still higher than my conclusion makes it; since a portion of space has entered the circle, which has not been taken into account.

Nevertheless, that the real space of the two figures in question is identical, the further developement of the subject will, I think, make evident.

> " Veritas nunquam perit."
> SENECA.

But, in order to proceed intelligibly, I shall find it necessary to distinguish for reference certain compartments of the diagram, generated by demarcating the square into its chief component parts triangular and circular. The utility of my nomenclature will, I hope, excuse its novelty and harshness.

> " Et vous, vous y venez d'un œil observateur,
> Admirer dans ses plans l'éternel créateur !"
> DELILLE.

SECTION V.

ANALYSIS OF THE DIAGRAM.

> "This does but rough-hew and design,
> Leaves Art to polish and refine."—BUTLER.

> " Quod superest, vacuas aureis mihi, Memmius, et te
> Semotum à cureis, adhibe veram ad rationem :
> Ne mea dona, tibi studio dispôsta fideli,
> Intellecta prius quàm sint, contempta relinquas."—LUCRETIUS.

1. THE central compartment, in the diagram, with concave sides and four extremities touching the circumference of the circle, is what I would denominate the *star*. It is a figure well worth our notice : although for luminosity it be not comparable, to the metaphorical guiding star of intellect that radiates in all circles, our political fourth estate, the PRESS ; nor yet to those supernal stars of the firmament, the beautiful and bright mute pilots of regal night,

> " That navigate a sea that knows no storms."
> COWPER.

The four concave corners that clasp the circle externally, are the star's exact counterpart severed into quarters. And if we allow scope to the luring freaks of fancy, the star may be regarded as the emblem of universal charity, with arms circled out on every hand to embrace a world, yet with extremes so delicately fine as to surpass, in minuteness, even the widow's mite.

2. The square that lies within the star, and reaches to its boundary lines, is intersected into eight triangles, which compartments, for distinction's sake, I would speak of, *par excellence*, as *trigons*. A square, when formed of two half trigons, may be termed a *semi-trigonal square*, from its construction.

3. The first of the square frames that accrue around the centre square constitutes it a fourth part of the full square;

> "On either hand
> Enlarged it spreads around."
> AKENSIDE.

The second frame is a second fourth; the third frame is a half. It may amuse,—if punning is permissible in mathematics,—

> "Et sermone opus est, modo tristi sæpe jocoso;"
> HORACE.

to remark, ere we pass on, how these universal frames regularly reduce all into *square*, by clinging to the square at their heart; emblem of rectitude, and a rare sample of *squaring* continually going on, and *differences* constantly arising, yet no *falling out* among the parties, or *breach* of the *piece*.

4. The central star, when augmented at its sides, by the four proximate segments, forms a square equal to half the full square; and the square that crosses it horizontally is a counterpart half.

5. Diagonal lines crossing the full square intersect it into *quatral* triangles; these are bisected into *octrals;* and these, again, into *sedecimals.*

6. Each octral triangle holds the space of a quarter star and segment; and, on assuming a similar segment in addition, becomes a *quadrant* of the circle.

7. In each sedecimal triangle pendant from the top line of the full square, may be observed a *rectangular parallelogram,* equivalent in area, as intersected, to two trigons and a *residual* part of another: for a portion of the third trigon has overlapped or run-over, as it were, emblem of good measure.

8. Two trigons, by thus overlapping each other, within the sedecimal triangle, form a *sub-trigon;* a *residual* is the portion remaining when the space of a sub-trigon is subtracted from a trigon, and, of course, it is not restricted to a specific form, but occurs in various shapes.

9. The two smaller rectangles formed in the rectangular parallelogram, by bisecting it with a perpendicular line, may be termed *panes.* Let the one, whose greater part lies within the circle, be known as the *inner-pane.*

10. Let the part cut out, at the peak of the central trigon of those that are within the sedecimal triangle, by the crossing of circumferent lines, be designated the *clip,* or *compasses*; further distinguishing its parts, as exteriorly cut, into the *apex,* and the *limbs*; and let the rest of the trigon be known as the *plus'd-residual.*

11. To discriminate the compartments composing the concave corners that engird the segments, the central part may, from its shape, be aptly termed the *mitre;* and the complemental parts extending on either hand of the mitre may be conceived of as the *wings.*

12. The middle portion of each wing, as cut off within the rectangular parallelogram, I shall name the *commuted-residual*; for, that it is equal to a residual, I must still contend. Besides, the very stucture of this rectangle shews that the addition to it of a trigon *plus* a sub-trigon, would form it into a square containing just four trigons; and it would be strange indeed, were lines, traversing the diagram so widely as do these intersecting lines of the transverse circles, to cut the square into compartments so nearly *aliquot,* or resemblant, but not quite so.

13. The commuted residual, when withdrawn from the parallelogram, leaves within, what we will term the *commuted trigons;* and these may be turned into semi-trigonal squares, by depriving each of its figure "Inigo," and adding the figure "Outigo" in lieu. A semi-apex and a semi-clip lie on each side of the commuted residual to complete the wing.

14. As an instance how sameness of value is maintained under change of form, it should be noticed how the transference of a trigon from one side of the rectangular parallelogram to the other, renders the parallelogram *lozenge*, in shape, and equilateral:

> "Some ever new diversity of grace."
> BROWN.

15. The longitudinal bisection of the lozenge exhibits to us counterparts of the *minor-segment*, and its *concave cap*. This cap will be found equivalent, in space, to a semi-trigon; and, consequently, each minor-segment must contain a semi-trigon, *plus* a semi-residual.

16. In our right hand sedecimal triangle pendant from the top line of the full square, a clip is shewn, incident on the sub-trigon formed within the peak of the central trigon, but with the extremities of each limb extruded into the residual of the trigon, so as to leave visible to us, in the sub-trigon, a *cave* or hollow space. That this hollow is equivalent to the space occupied by the extruded portions of the limbs of the clip, *plus* a semi-apex,—we will now institute a comparison of compartments to make evident.

> "It is a weary interlude."
> KING.

SECTION VI.

COINCIDENCE OF EXTERNALS WITH INTERNALS.

> "What I'm to prove by argument."
> "Verily,
> The point seems very plain to me."—BUTLER.

THE subject of our comparison shall be a concave corner of the square, and its proximate segment. These two compartments, it must be borne in mind, are, together, the equivalent of an octral triangle, whose contents are, in terms of the triangle, exactly ten trigons, *plus* two residuals.

Let us place the smaller of these compartments, within the other, as it were, by successively laying portions of the exterior figure, upon similar quantities in the interior one.

In the first introsusception, the mitre from without will cover the two semi-mitres that are within. Then, upon the two inner trigons, we will place, from the wings, their two commuted-residuals, their four semi-clips, and two out of their four semi-apices,—since these spaces computed together constitute two trigonal equivalents. We have now remaining outside our segment only two semi-apices. We may safely infer, then, that these semi-apices are the equivalents of the *pluses* of the hollows, in the two internal *plus'd*-residuals; and, consequently, that each is equal to a *plus;* since, when put into the segment, the only portion of that figure that will remain undoubled, will be the exact equivalent of two residuals; and the gross internal contents of the loaded segment will have become equal to the measure of the octral triangle, whose compartments we have thus compared. And, observe, further! the extent of the over-lay being a space of exactly five trigons, proves that five trigons are the correct measure of an external concave corner, or fourth part of a star. It proves, besides, that the full square, *minus* a star or twenty trigons, is the content of the circle: and corroborates the statement at paragraph 15, of Section V, Analysis of Diagram, that the caps within the lozenge are each the equivalent space of a semi-trigon.

SECTION VII.

CAPACITY OF CERTAIN COMPARTMENTS.

"For brevity is very good
When w' are or are not understood."—BUTLER.

’Αλλ’ οἱ φρονοῦντες εὖ, κρατοῦσι πανταχοῦ.—SOPHOCLES.

INNUMERABLE inferences, as to the capacities of spaces, might be drawn from an intelligent inspection of the diagram; but, a few only it must suffice to mention; since "brevity is the soul of wit," and never so brief, nor so witty either—if the subject is a dull one,—as when things are left to speak for themselves.

Hæcce memento loqui quasi voce silentium apertâ.

The content of the first frame around the centre square is the measure of twelve trigons, *plus* four residuals; of the second frame, it is just eight trigons more.

The mitres each contain three trigons *minus* an apex; towards this quantity each right-lined, wedge-shaped, compartment within the same figure, furnishes the space of a semi-trigon, *plus* a semi-apex.

A wing is the equivalent of a trigon, *plus* a semi-apex.

Were another circle to be inscribed close around the corners of the first frame, its area would equal only half the content of the circle that we see inscribed around the second; whilst one similarly drawn around the third frame, or full square, would hold double the content of the circle engirding the second; and all analogous compartments of these various circles would vary in similar proportions.

Eighty trigons, or four stars, in addition to sixteen residuals, equal the full square; since sixty trigons, or three stars, in addition to sixteen residuals, equal the circle within it.

The space of a trigon, *plus* one fifth of a residual, is equal to one fifth part of the sedecimal triangle, and amounts, in the scale of the square of 24, to 7 units, *plus* $115\frac{1}{5}$ of the 576 fractional parts.

The space of thirty five trigons will be found equivalent to 216 units, *plus* a subter-trigon of the square of 24; and, of course, dividing the latter amount by 35, reproduces the content of a single trigon.

How harmonious, and beautiful, to the eye of taste, are these samples of symmetries! Inspection of the diagram, and fresh intersections of the compartments, would furnish others in abundance:

"O Nature! a' thy shows an' forms
To feeling pensive hearts hae charms."
BURNS.

In the sedecimal triangle, even the deficiency of a sub-trigon to complete the uniformity of six trigons, is no more to be considered a defect or blemish, than were the visual and vocal deficiences of the sweet lady so unfortunately constituted—of course by a freak of Nature—as not to be able to scold, blemishes, in the appreciation of the poet who has recorded them:

"Hop'st thou to frown with eyes that smile,
And chide, with tones that but endear thee?
Thou may'st do all things ere revile,—
And all things I,—ere learn to fear thee."
BROWN.

For my part, I gratefully acknowledge that this overlapping or extruding of a part of a trigon, has afforded me the clue, by which I arrive at an accurate gauge of the whole figure:

"Think not the things most wonderful
Are those beyond our ken,—
For wonders are around the paths,
The daily paths of men."
HAWKSHAW.

SECTION VIII.

CONTEST OF INIGO AND OUTIGO.

> "O tu, che leggi, udirai nuovo ludo."—DANTE.
>
> "A battle whose great scheme and scope
> They little cared to know."—MILNES.
>
> "For if they are the same, by course,
> Neither is better, neither worse."—BUTLER.

THE seeming adversaries, Inigo and Outigo, will now try a short contest for the supremacy. On one of the inner panes of the rectangle, we start them fairly to essay—

> "The onset's doubtful strife."
> BROWN.

The straight line that from the inner pane cuts off the square formed of two semi-trigons, leaves us assured of a semi-residual to commence with. Conceive, then, Outigo to leave the semi-trigonal square and enhance this space, whilst Inigo directs his march into the circle, and attaches to that square, taking with him for ally just one particle more. The space under regard will then contain a semi-residual, *minus* a particle; and the wedge of the pane, which before held but a semi-trigon, *plus* a semi-apex, will receive the particle as a supernumerary.

How stand the wings affected? they have each become equivalent to the space of a trigon, *plus* a semi-apex, *minus* two particles. We will again fill the segment with the quarter star of the concave corner. Mitre covers mitre. The equivalents of trigon-spaces are laid over the trigons. But alas! our remaining pair of semi-apices are each deficient, by two particles, of space enough to fill the hollows of the *plus'd*-residuals; and, on summing up, the segmental contents are found to be just four particles short of the measurement of the octral triangle.

But some one may say "yet the wedges have each been increased by a particle, thus enhancing the semi-mitres." "True," I reply, "but that circumstance does not alter the discrepancy; for remember! the *external* mitre has overlaid and balanced the *internal* one."

Let the contention be reversed, and try the effect of an auxiliary particle accruing to Outigo. Since there is an Outigo in each pane, the commuted residual becomes thus enhanced by two particles; and the wing contains these two in addition to its trigon, *plus* a semi-apex. Again the mitre enters the segment; the trigons within become overlaid; and, this time, even the *pluses* of the hollows are covered by the semi-apices; but, alack! alack! four particles remain in excess, beyond the forces wanted to balance the octral triangle.

Peace must ensue; for the one encounter ended by shewing four particles *too little*, the other with shewing four particles *too much*, of the space required to prove that a quarter star and segment are, taken together, equivalent to the space occupied by the octral triangle. Wherefore I think we may fairly conclude

> "Medio tutissimus ibis:"
> OVID.

and that amity may endure between the parties henceforth; so that, far from Inigo and Outigo's being regarded by us as contending whether the one is really superior to the other in extent of space, they must unavoidably be concluded to be in that respect perfectly equal: "*quod erat demonstrandum.*"

The only argument of paramount importance bearing on the question of the circle's real content, is, as appears to me, this determination of ours as to the sameness of space occupied by these peculiar compartments; and therefore it is that I earnestly caution the reader not hastily to form his conclusion from the visible inaccuracy of their representation in the diagram; but from carefully reasoning on the consequences that must necessarily result to the circle as a whole, and its constituent figures calculated as ascertained parts, if we disturb their amity, by making them, in our estimate of their respective content, diverse from each other, in the least possible respect.

SECTION IX.

MEASURE OF THE TRIGON AND ITS PARTS.

"Oh! that I could enucleate!"—Butler.

"Give, since my path is doom'd—not wholly dark,
One little vista of my way to mark!"—Brown.

"I wol you techen pleinly the manere
How I can worken in philosophie."—Chaucer.

AFTER distinguishing, by original names, the several compartments into which the intersected circle and its complemental corners resolve the square figure; I have endeavoured to prove, to the satisfaction of the examiner, that certain diversities of form, generated by the lines of intersection, are nevertheless perfectly equal to each other in respect of areal space.

If my reasoning has been successful in shewing that the trigon, the commuted-trigon, and a pair of concave caps are identical as to space, I have shewn, at the same time, how circular and square forms correspond together, so as to become equally the subjects of approximate measurement by numeral notation; even though we admit the trigon to be a space incommensurable by the gauging square unit.

I have now to develope a method by which the correct measure of the trigon and its natural constituent parts may be evolved for numeral expression, to whatever degree of accuracy the calculator may be pleased to pursue the computation of it:

"Destati, pigro ingegno, da quel sonno,
Che par che gli occhi tuoi d'un vel recopra,
Onde veder la verità non ponno."
Lorenzo de' Medici.

This method which, by long beating about the bush, I have been so fortunate as to discover, consists of a series of constructive analyses of the sedecimal triangle. What I adopt for my elucidation of the subject, is the square of 24, which gives us 576 as the content of the full square, and, consequently, 36 as the content of the aforesaid triangle.

The sedecimal triangle is first of all divided geometrically,—as is shewn in the diagram,—into five trigons, and a residual; then, the trigon is similarly subdivided into five sub-trigons, and a sub-residual; the sub-trigon, similarly, into five subter-trigons, and a subter-residual; and so on, successively:

"No unregarded star
Contracts its light,
Into so small a character."
Habington.

"Nous avons beau enfler nos conceptions," we may carry forward the operation, apparently, *ad infinitum*, like the endless divisibility of matter; applying, as we proceed, whatever names we may conceive suitable to express the descending series of minuter corresponding compartments that will result, until from primeval chaos we evolve its primal element,

"A nameless and eternal thing—"
Byron.
"No thought can figure, and no tongue declare:"
Prior.

so true is the observation of a French Naturalist of great research—"à mesure que l'homme s'approche des éléments de la nature, les principes de sa science s'évanouissent."

And how humiliating to the pride of scholarship, is the consideration, that though we may be warranted in boasting of our acquisitions of *knowledge;* we must ever speak but of our pursuits of *science!*

"Noi veggiam, come quei ch'ha mala luce,
Le cose, disse, che ne son lontano;
Cotanto ancor ne splende 'l sommo Duce."
Dante.

SECTION X.

APPLICATION TO A CASE.

"Let us glance for a moment, 'tis well worth the pains!"—**Lowell.**

"Now low—and lower yet—and yet more low,—
Like Echo sighing to some lover's woe."—**Brown.**

On inspecting the diagram, we may perceive that if, from the sum to which six trigons amount, we deduct the numeral content of the sedecimal triangle, viz. 36, the remainder will be the content of the sub-trigon.

If, from the sum to which six sub-trigons amount, we deduct the content of a trigon, the remainder will be the content of the subter-trigon.

Again, if, from the sum of six subter-trigons, we deduct a sub-trigon, the remainder will be an infrâ-subter-trigon; and the analysis may thus go on advancing—

"Small by degrees, and beautifully less,—"

until we are weary, or satisfied with the extent of our approximation; for, that the lowest result at which we can arrive, with numeral characters, will be, after all, but an approximation, a few trials will serve to make apparent; since, either one or more complemental units continually want adding to the fraction of the trigon, to enable our calculation to proceed; or, the last unit added will so overstate this fraction, that the amount of the residue, at the extremity to which our analysis shall have been carried, when, as usual, it be multiplied by six, will be conspicuously too great, and will continue producing a greater and greater discrepancy and distortion, the longer the process is carried on.

"Nature, ô séduisante et sublime déesse,
Que tes traits sont divers!"
Delille.

Having myself, by successive trials, eliminated the correct fractional numerators in the fraction of the trigon, as far as unto the twenty second power of 576, I am enabled feelingly to endorse to the operator the words of the indefatigable lexicographer "Totius Latinitatis,"

"Expertus disces quàm gravis iste labor!"
Forcellinus.

Certainly mine has been little more than half a tithe of his forty years of labour, yet I have often paused from weariness. Nature is the only untiring operator,

"Working in secret its resistless spell
Witching the heart unutterably well."
M.S. of "**Alzara.**"

Not, however, to unduly weary my readers, I will here adduce only a specimen of the calculation, with the remainders arising from the invisible portion of the fraction stated; since it is necessary to take these into the account, to ensure exactness. In my concluding Section, I will introduce the results in respect of the leading compartments of the figure, that any one, so disposed, may verify my computation *in extenso*, and gain—

"Increase of power from lore of dullest things."
Brown.

First, however, let us examine what my method makes of the computation of the circle's area, that is current in the schools, and which originated, I believe, with Ludolf Van Ceulen; and as so extraordinary an exploit was his performance regarded, that we are told it was recorded in thirty-six places of figures cut on his *tombstone* in St. Peter's Church-yard, at *Leyden*. But alas for human vanity!

"The best laid schemes o' mice and men
Gang aft ajee."
Burns.

SECTION XI.

AREA OF SCHOOLS AND AUTHOR CONTRASTED.

"Palmam qui meruit ferat."

"Ὅτῳ δὲ μὴ τάδ' ἐστὶν ἐν γνώμῃ φίλα,
Κεῖνός τ' ἐκεῖνα στεργέτω κἀ'γὼ τάδε.—SOPHOCLES.

"EXPERIENCE teacheth many things, and all men are his scholars:
Yet is he a strange tutor, unteaching that which he hath taught."—TUPPER.

IF the area of the circle within the square of unity, as given by the schools in the decimal .785398163397448309615660845969875 &c.—and carried on, it is stated, by a most assiduous calculator to the amazing extent of no less than 128 decimal places without terminating,—be taken by us, and reduced into terms of the fraction of successive descending powers of the square of 24, its measure will be found to correspond to the following notation of it—

.452 224 150 213 131 25 63 &c.

Let us deduct 432 from this amount, being three-fourths of five hundred and seventy-six, the sum of the fractional parts of our square; all that will then remain in the circle, will be the presumed value of the four residuals of the construction, amounting to—

.20 224 150 213 131 25 63,

and giving us for a single residual—.5 56 37 341 176 438 159.

On deducting this residual from the sedecimal triangle of 36 parts, our remainder, left for the five trigons of its contents, comes out— .30 519 538 234 399 37 416,

giving us for a single trigon— .6 103 568 277 195 7 313.6,

or, for six trigons— .37 47 530 512 18 45 154.

To get the sub-trigon, let us deduct 36 parts from the above sum; and, on applying to this sub-trigon, the method of analysis which I have indicated, observe the extravagant result, in the distortion of amount that would speedily ensue, or, rather, commence;

"For there is nothing in the earth so small that it may not produce great things,
And no swerving from a straight line, that may not lead eternally astray:"
TUPPER.

.1	47	530	512	18	45	154
.6	287	305	192	108	271	348
.6	103	568	277	195	7	313.6
.0	183	312	490	489	264	34
.1	525	149	65	56	432	204
.1	47	530	512	18	45	154
.0	477	194	129	38	387	50
.4	560	13	198	232	18	300
.0	183	312	490	489	264	34
.4	376	276	283	318	330	266.

If, then, 432 *plus* 4 residuals,—as my resolution of the figure into its own terms shews it must be,—be really the exact content of the circle generated within the square of 24; it is manifest the schools are wrong in their computation of it.

"Interdùm vulgus rectum videt; est ubi peccat."
HORACE.

It remains to apply the same process of analysis to my own computation of the area, which is—

.452 269 171 99 257 113 557 &c.

deducting 432 parts leaves us for the 4 residuals—

.20 269 171 99 257 113 557,

and gives us for a single residual— .5 67 186 456 496 172 283.

The deduction of this residual from the sedecimal triangle of 36 parts, leaves remaining, as the sum of the five trigons— .30 508 389 119 79 403 292,

giving us for a single trigon— .6 101 423 254 131 80 404,

or, for six trigons, *minus* 36 parts, equal to the sub-trigon—

.1 34 236 373 210 484 120.

The operation, with the proper invisible remainders taken into the account, will then stand as at the close of this Section, and speaks for itself;

> "Cùm tu horum nihil refelles, vincam scilicet:"
>
> Terence.

.1	34	236	373	210	484	120	3
.6	206	267	512	113	25	147	
.6	101	423	254	131	80	404	
	104	420	257	557	520	319	4
.1	52	218	395	467	243	190	
.1	34	236	373	210	484	120	1
	17	558	22	256	335	69	5
	107	468	134	387	282	419	
	104	420	257	557	520	319	1
	3	47	452	405	338	99	5
	18	286	412	129	301	23	
	17	558	22	256	335	69	1
		304	389	448	541	529	5
	3	100	34	389	371	299	
	3	47	452	405	338	99	1
		52	157	560	33	199	4
		313	371	480	200	46	
		304	389	448	541	529	1
		8	558	31	234	92	3
		53	468	188	252	555	
		52	157	560	33	199	1
		1	310	204	219	355	2
		9	134	74	165	404	
		8	558	31	234	92	
			152	42	507	312	1
		1	336	257	165	145	
		1	310	204	219	355	1
			26	52	521	365	4

26	52	521	365	4
156	317	249	466	
152	42	507	312	
4	274	318	154	2
26	495	181	350	
26	52	521	365	1
	442	235	560	1
4	350	263	481	
4	274	318	154	
	75	521	327	
	455	249	234	
	442	235	560	
	13	13	250	3
	78	80	351	
	75	521	327	
	2	135	24	3
	13	234	147	
	13	13	250	
		220	473	1
	2	172	535	
	2	135	24	1
		37	510	5
		227	185	
		220	473	
		6	288	1
		39	1	
		37	510	1
		1	66	2
		6	398	
		6	288	
			110	1
		1	85	
		1	66	1
			18	5
			113	
			110	
			3	1
			19	
			18	1
			0	3.

SECTION XII.

CONCLUDING REMARKS.

> "In mid descent he paus'd."—Brown.
>
> "Thoughts, whither have ye led me? with what sweet
> Compulsion!"—Milton.
>
> "Ἧττον ἄν ἁμάρτοις, εἰ νοεῖς τὶ ἀγνοεῖς."—Scaliger.

It is evident, by the diagram, that the sedecimal triangle is equal to five trigons, *plus* a residual; therefore (12 × 36) or 432, is the sum of 60 trigons, *plus* 12 residuals; and, consequently, (18 × 432) or 450, is the sum of 62½ trigons, *plus* just 12½ residuals; and the surplus of the circle's contents beyond 450, must then clearly be equal to 1½ sub-trigons, *plus* a sub-residual; or—to state it by numerals, in the fraction of the descending powers of 576,—to 2.269 171 99 257 113 557 487 558 344 336 412 0 538 102 541 545 385 213 320 383 184, *minus* 20 of the unevolved complement; which, be its numeral value what it may, is certainly less than a unit of the last stated denomination of the fraction; an approximation to the veritable contents of the circle that—as I have before explained,—may be carried still more close, at the choice of the industrious operator; and, surely, we may consider this a satisfactory conclusion to arrive at:

> "Minùs sæpè pècces, si scias quid nescias."
> Syrus.

In like manner, by the diagram, we find the four concave corners,—that, added to the area of the circle, give the area of the square,—equal to a star, or 20 trigons; which sum of trigons, stated in the fraction of the descending powers of 576, amounts to 123 .306 404 476 318 462 18 88 17 231 239 163 575 37 473 34 30 190 362 255 192 392, *plus* 20 of the unevolved complement.

My resolution of the full tetragon, or square, into terms of the sedecimal triangle, shows its full content to be 80 trigons, *plus* 16 residuals. These residuals, reduced into terms of the trigon, become 13 trigons, *plus* 1 sub-residual, 3 subter-trigons, and 1 subter-residual.

Let us state these items at their numeral value in the square of 24. If, upon being summed up together, they make the total amount, 576,—which is the sum of the number 24 when squared,—the accuracy of my method of analysis will be thereby confirmed, and illustrated.

																							COMPLEMENTS.		
574.	245	212	27	100	17	142	6	80	528	507	503	226	31	500	302	198	368	14	553	406	210	+	93	=	93 trigons.
	505	392	115	228	539	376	505	550	442	27	251	462	89	91	382	359	126	453	378	325	238	—	29	=	1 sub-residual.
	314	109	197	521	409	383	174	91	206	536	140	571	54	323	322	447	136	462	476	291	330	+	105	=	3 subter-trigons.
	86	438	235	301	185	249	465	428	550	80	255	468	400	236	144	146	520	220	319	128	374	—	169	=	1 subter-residual.
576.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0		

Every mathematician is aware that in the square of unity,—since the star and its cognate circle exist in ratio together,—it follows, as a necessary consequence, that just as the multiplication of the areal unit of the square, by four, produces its perimeter; so, the multiplication of the areal fraction representing the contained circle, by four, must produce the measure of that circle's periphery, or circumference. Hence, though my remarks have been directed to elucidate the areal measurement only, they can readily be applied to elucidate the measurement of circumferent lines also.

And, again, since by extracting the square root of any circle's content, we should get the correct measure of the side of a square whose content would equal the area of that circle, the practical use of knowing the certain contents of the star and circle, becomes apparent. These compartments, when taken together, and lowered one step in the scale, are the equivalent of unity; if, therefore, the length of the side of any square be squared, and the result be used as a multiplier of the evolved fractions that represent either of them, the product will be, respectively, the area of the star, or circle, pertaining to the square whose squared side we have used as multiplier.

But my course must be stayed; revelling at times amid "the wild sublimities of thought," I have quaffed most pleasantly the requiting gale,—

> "what Science breathes
> Of calm, mild rapture, on the wakeful soul
> That scans with self-fix'd thought profoundest things."
> Brown.

In contemplating the result of my investigations, it may be deemed no slight cause for gratulation, to have shewn that, henceforth, no eager student need, through blind ignorance, plunge into perplexity, and waste precious time, in the futile attempt to discover a method of representing incommensurable spaces to a nicety, by numeral notation;

> "For what's impossible can't be,
> And never, never comes to pass!"
> Colman.

at the same time, as far as there can possibly be a vestige of utility in the calculation, my method shews clearly how approximation may be correctly pursued. My motto has been "*labor omnia vincit,*" and after a most unconscionable expenditure of time and thought upon the subject, I incline to think I have deserved well of the community at large:

> "Let all the ends thou aim'st at be thy country's."
> Shakspeare.

My researches have been restricted to a plane, and its superficial dimensions only. In this direction of space, it is curious to observe how interminable a continuity of stars and circles—

> "Turn round to square, and square again to round:"
> Pope.

or, to speak more elementally, how endless a succession of stars and double segments might be mapped out on every quarter of a nucleate circle. And, what is well worth remark, in any defined square of space, the same ratio subsists between these two kinds of elemental figures, whatever may be their proportions, or number, provided they are *aliquot*, or equal to the whole when summed up together. But "wave on wave with flooding force throng the shoals of thought," as saith the philosopher; and to the students of atomic theories, or of the invisible interstitial capacities of bodies, I would say, with the poet,

> "Press on! for it is godlike to unloose
> The spirit, and forget yourself in thought."
> Willis.

Such individuals will do well to study the cubicular and spherical dimensions and directions of space; in which similar successions,—but, instead of planes,—of cubicular stars, and segmental lobes or spindles, will be found expatiating, as it were, through the universe, far far beyond the ability of the sciolist in science to pursue them,—

> "unskill'd to range
> From orb to orb, where Newton leads the way;
> And view with piercing eyes the grand machine,
> Worlds above worlds, subservient to His voice,
> Who, veil'd in clouded majesty, alone
> Gives light to all."
> Somervile.

Like the poor spider whose ultimate success in climbing,—after many fruitless attempts,—is said to have aroused the failing spirit of the Bruce to persevere and vanquish, I, too, have encountered my many many failures, in arriving at a satisfactory solution of this remarkable, and most interesting Problem. The narrative of my erratic course might read like a fairy-tale, were I to recount the hosts of fancies, and analogies, amid which "thought and her shadowy brood" have run riot, in the progress of my researches,

> "False as light visions that in slumber wave,
> Yet O! how real in the joy they gave!"
> Brown.

but, for the present, I forbear to enlarge.

> "I have not now ne mo to sey."
> Chaucer.

ANALYSIS OF THE SEDECIMAL TRIANGLE, *IN EXTENSO.*

1	6	.101	423	254	131	80	404	62	0	501	98	209	460	433	513	145	404	412	421	185	326	250	+ 1 COMPL.
2	37	.34	236	373	210	484	120	372	5	127	14	106	460	299	199	298	124	172	223	537	230	348	+ 6 DITTO.
	6	.206	267	512	113	25	147	504	81	186	85	64	459	68	45	61	169	458	191	344	231	360	
	6	.101	423	254	131	80	404	62	0	501	98	209	460	433	513	145	404	412	421	185	326	250	
3		.104	420	257	557	520	319	442	30	260	562	430	574	210	107	491	341	45	346	158	481	110	
	1	.52	218	395	467	243	190	348	182	413	496	281	566	109	71	69	318	273	349	377	7	84	
	1	.34	236	373	210	484	120	372	5	127	14	106	460	299	199	298	124	172	223	537	230	348	
4		.17	558	22	256	335	69	552	177	286	482	175	105	385	447	347	194	101	125	415	352	312	
		.107	468	134	387	282	419	433	488	569	13	475	58	10	381	356	13	31	178	189	387	144	
		.104	420	257	557	520	319	442	30	260	562	430	574	210	107	491	341	45	346	158	481	110	
5		.3	47	452	405	338	99	567	458	308	27	44	59	376	273	440	247	561	408	30	482	34	
		.18	286	412	129	301	23	526	447	120	162	264	357	530	490	338	335	490	144	185	12	204	
		.17	558	22	256	335	69	552	177	286	482	175	105	385	447	347	194	101	125	415	352	312	
6			304	389	448	541	529	550	269	409	256	89	252	145	42	567	141	389	18	345	235	468	
		.3	100	34	389	371	299	422	466	152	384	536	361	294	257	523	274	30	111	344	262	504	
		.3	47	452	405	338	99	567	458	308	27	44	59	376	273	440	247	561	408	30	482	34	
7			52	157	560	33	199	431	7	420	357	492	301	493	560	83	26	44	279	313	356	470	
			313	371	480	200	46	282	46	219	419	75	83	83	480	498	156	266	525	153	412	516	
			304	389	448	541	529	550	269	409	256	89	252	145	42	567	141	389	18	345	235	468	
8			8	558	31	234	92	307	352	386	162	561	406	514	437	507	14	453	506	384	177	48	
			53	468	188	252	555	117	388	13	401	490	137	208	323	162	88	419	160	1	486	288	
			52	157	560	33	199	431	7	420	357	492	301	493	560	83	26	44	279	313	356	470	
9			1	310	204	219	355	262	380	169	43	573	411	290	339	79	62	374	456	264	129	394	
			9	134	74	165	404	423	553	438	263	562	165	15	306	474	375	520	434	433	202	60	
			8	558	31	234	92	307	352	386	162	561	406	514	437	507	14	453	506	384	177	48	
10				152	42	507	312	116	201	52	101	0	334	76	444	543	361	66	504	49	25	12	
			1	336	257	165	145	122	54	313	30	3	276	460	365	381	438	401	144	294	150	72	
			1	310	204	219	355	262	380	169	43	573	411	290	339	79	62	374	456	264	129	394	
11				26	52	521	365	435	250	143	562	5	441	170	26	302	376	26	264	30	20	254	
				156	317	249	466	308	349	287	492	34	343	444	159	87	528	158	432	180	122	372	
				152	42	507	312	116	201	52	101	0	334	76	444	543	361	66	504	49	25	12	
12				4	274	318	154	192	148	235	391	34	9	367	290	120	167	91	504	131	97	360	
				26	495	181	350	1	314	262	42	204	57	477	13	145	426	551	145	211	9	432	
				26	52	521	365	435	250	143	562	5	441	170	26	302	376	26	264	30	20	254	
13					442	235	560	142	64	118	56	198	192	306	562	419	50	524	457	180	565	178	
				4	350	263	481	276	385	132	338	38	3	113	496	210	305	268	439	509	511	492	
				4	274	318	154	192	148	235	391	34	9	367	290	120	167	91	504	131	97	360	
14					75	521	327	84	236	472	523	3	569	322	206	90	138	176	511	378	414	132	
					455	249	234	506	268	533	258	23	537	206	84	541	253	485	189	544	181	216	
					442	235	560	142	64	118	56	198	192	306	562	419	50	524	457	180	565	178	
15					13	13	250	364	204	415	201	401	344	475	98	122	202	536	308	363	192	38	
					78	80	351	458	76	188	58	105	340	547	13	158	65	339	123	452	0	228	
					75	521	327	84	236	472	523	3	569	322	206	90	138	176	511	378	414	132	
16					2	135	24	373	415	291	111	101	347	224	383	67	503	162	188	73	162	96	
					13	234	147	514	189	19	91	33	356	195	570	407	139	397	552	439	397	0	
					13	13	250	364	204	415	201	401	344	475	98	122	202	536	308	363	192	38	
17						220	473	149	560	179	465	208	11	296	472	284	512	437	244	76	204	538	
					2	172	535	323	481	502	488	96	69	52	530	557	196	320	312	458	77	348	
					2	135	24	373	415	291	111	101	347	224	383	67	503	162	188	73	162	96	
18						37	510	526	66	211	376	570	297	404	147	489	269	158	124	384	491	252	

18	37 510 526 66 211 376 570 297 404 147 489 269 158 124 384 491 252
	227 85 276 398 1 7 533 543 58 121 3 56 463 373 172 5 68 360
	220 473 149 560 179 465 208 11 296 472 284 512 437 244 76 204 538
19	6 288 126 413 514 68 335 46 400 414 347 526 511 503 504 439 398
	39 184 79 204 4 282 280 100 183 359 281 91 43 48 334 84
	37 510 526 66 211 376 570 297 404 47 489 269 158 124 384 491 252
20	1 66 234 112 569 34 287 558 272 35 446 12 33 18 339 418 408
	6 398 253 101 534 206 575 470 480 214 372 72 198 111 310 208 144
	6 288 126 413 514 68 335 46 400 414 347 526 51 503 504 439 398
21	110 126 264 20 38 240 424 79 376 24 21 262 183 381 344 322
	1 85 182 432 21 254 292 240 477 528 145 52 421 525 561 339 204
	1 66 234 2 569 34 287 558 272 35 446 2 33 8 339 418 408
22	18 524 3 9 28 220 4 258 205 492 275 40 388 507 22 496 372
	13 267 187 194 168 26 398 83 74 499 268 29 164 79 99 504
	110 126 264 20 138 240 424 79 376 24 121 262 183 381 344 322
23	3 140 499 174 29 361 550 3 274 475 146 342 556 373 331 182
	9 269 115 468 77 443 420 20 496 547 303 329 459 513 259 516
	18 524 319 128 220 4 258 205 492 275 40 388 507 221 496 372
24	320 372 339 533 439 16 391 4 272 62 516 528 291 339 144
	3 195 507 311 322 331 394 42 26 481 401 221 291 21 307 288
	3 140 499 174 29 361 550 3 274 475 146 342 556 373 331 182
25	55 8 137 292 545 420 38 328 6 254 454 310 223 552 106
	330 49 249 29 394 216 231 240 38 376 423 34 9 433 60
	320 372 339 533 439 161 391 4 272 162 516 528 291 339 144
26	9 252 485 71 531 54 416 235 342 213 482 81 476 93 492
	56 365 30 43 306 328 94 26 326 31 3 514 552 563 72
	55 8 137 292 545 420 38 328 6 254 454 310 223 552 106
27	1 356 469 138 336 484 155 509 319 452 135 204 329 10 542
	9 412 5 255 293 25 359 77 190 409 236 75 246 65 372
	9 252 485 71 531 54 416 235 342 213 482 181 476 93 492
28	160 26 83 337 546 518 5 7 424 95 329 469 345 547 456
	384 57 525 299 40 233 226 242 21 250 513 347 406 432
	1 356 469 38 336 484 155 509 3 9 452 35 204 329 10 542
29	27 264 386 538 493 77 292 498 145 115 309 18 395 466
	164 436 7 353 78 465 29 109 295 7 126 2 70 492
	160 26 183 337 546 518 517 424 195 329 469 345 567 456
30	4 409 410 5 107 522 87 261 99 363 232 342 99 36
	28 154 156 91 71 252 524 415 21 452 243 325 18 216
	27 26 386 538 493 77 292 498 145 5 309 8 395 466
3	465 345 128 154 175 231 492 452 336 510 306 98 326
	4 489 343 93 349 476 239 76 411 293 183 0 39 228
	4 409 410 15 107 522 87 261 99 363 232 342 99 36
32	79 509 78 241 530 5 39 3 505 526 343 516 192
	479 75 494 299 301 334 45 143 155 2 9 335 218 0
	465 345 128 154 175 231 492 452 336 510 306 198 326
33	13 406 366 145 126 02 28 266 394 345 29 19 250
	82 135 469 295 181 37 194 448 63 342 174 16 348
	79 509 178 241 530 151 391 311 505 526 343 516 92
34	2 202 291 53 226 46 379 136 133 39 406 76 156
	14 63 18 320 208 465 547 241 226 46 133 481 360
	13 406 366 145 126 102 128 266 394 345 29 19 250
5	232 228 175 82 363 418 550 407 277 104 462 110

35	232 228 175 82 363 418 550 407 277 104 462 110
	2 242 217 474 495 454 209 424 140 511 52 469 84
	2 202 291 53 226 461 379 136 133 391 406 176 156
36	39 502 421 268 568 406 288 7 119 222 292 504
	239 136 224 461 532 135 0 43 140 183 29 144
	232 228 175 82 363 418 550 407 277 104 462 110
37	6 484 49 379 168 292 25 211 439 78 143 34
	41 24 297 547 435 24 152 118 330 469 282 204
	39 502 421 268 568 406 288 7 119 222 292 504
38	1 97 452 278 442 193 440 111 211 246 565 276
	7 10 410 520 350 10 337 92 116 329 512 504
	6 484 49 379 168 292 25 211 439 78 143 34
39	102 361 141 181 294 311 456 253 251 369 470
	1 39 439 271 513 39 142 434 368 357 490 516
	1 97 452 278 442 193 440 111 211 246 565 276

The preceding analysis commences with a statement, by numerals, of the content of the trigon, and ceases with the numeral content of the thirty-ninth triangle of the series of minuter triangles regularly evolved from it. This ultimate quantity, it will be perceived, is too small in amount for the process of analysis to proceed further. But if a single unit be super-added to the lowest denomination in the fraction of the trigon, the following *ending* shews the result in the thirty-eighth, thirty-ninth, and fortieth triangles; their content being thereby rendered evidently too large for the process longer to proceed without a visible excess distorting it; so that it is made obvious, on inspection, that the numeral value, of the *complement* wanting to complete the trigon's exact content, is *less than a unit* of the lowest denomination of the fraction that we commenced with :—

	{ 5 343 11 252 193 69 113 52 149 540 114 = + 1 }
	{ 1 97 452 278 442 193 440 111 211 246 565 276 = 250 }
38	1 103 219 290 118 386 509 224 263 396 529 390 = 251
	{ 31 346 169 82 53 463 413 291 428 270 359 = + 1 }
	{ 102 361 141 181 294 311 456 253 251 369 470 = 250 }
39	134 131 310 263 348 199 293 545 104 64 253 = 251
	1 229 213 134 429 362 45 35 391 48 386 366
	1 103 219 290 118 386 509 224 263 396 529 390
40	125 569 420 310 551 111 387 127 227 432 552
	1 179 538 219 137 427 94 19 188 214 293 432.

THE RESULT IN THE DECIMAL SCALE.

DIMENSIONS OF SQUARES.

Side.	Area.	Perimeter.
1	1	4
12	144	48
24	576	96

DIMENSIONS OF THE CIRCLE WITHIN THE SQUARES.

Diameter.	Area.	Circumference.
1	.785533905932 +	3.142135623730 +
12	113.116882454314 +	37.705627484771 +
24	452.467529817256 +	75.411254969542 +

The Star = 123.532470182743 +

576 = the Square of 24.

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